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colourful Mathematics 5

FIND THE ACTIVATION CODE IN THE BOOK!
Digital EXTRAS

MOZAIK

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Mathematics

textbook

5

Mozaik Education – Szeged, 2019

That's the way you use the book

This textbook helps you review and understand topics covered in class, as well as helping you learn by yourself. The importance of each part of each lesson are covered on this sample page.

Lesson title

1

Each lesson starts with an introductory image and a related question, exercise or thought. Read and understand this! If you use your book in class, discuss the image, think about more examples, or act out a similar situation. If you are using your book at home, think about, how the image and the related proplem are linked to what you learned in class.

5

You can read about concepts you have already learned, interesting facts, questions, and explanations on the side of the page. Think about them!

2

Example

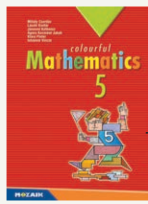
The pink fields contain sample problems and solutions. Read the question and try to solve the problem on your own first.

Solution

Compare your solution with the one in the book. It is possible that you arrived at the same solution by different means. Even if this is the case, look at the solution in the book, because it shows a method that you will have to use often to solve similar problems.

The samples also show you, how to write down a clear and concise solution to an exercise.

6



Solving the exercises in the workbook can help you understand and practice the material learned in class.

3

The normal text summarizes and explains the **essence of the lesson.**

The green fields contain important rules and the definitions of new concepts. Remember these!

Exercises

4

1. Solving the exercises after the lesson allows you to practice the new materials and methods learned in class.

*2. Problems marked with a * may require a clever solution.

7

Quiz

The quiz at the end of the lesson is not a usual mathematics exercise. We hope you enjoy solving it successfully.

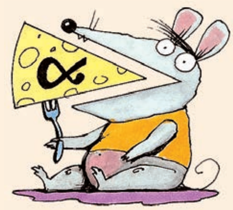
Table of Contents

Natural numbers

Geometry

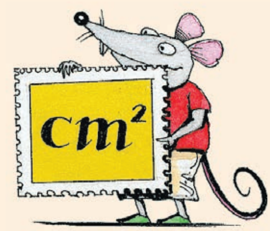
Measurement, statistics

Angles

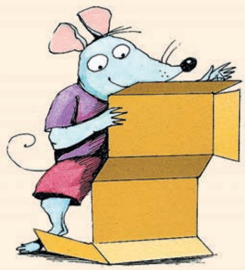


Fractions

The rectangle



The cuboid



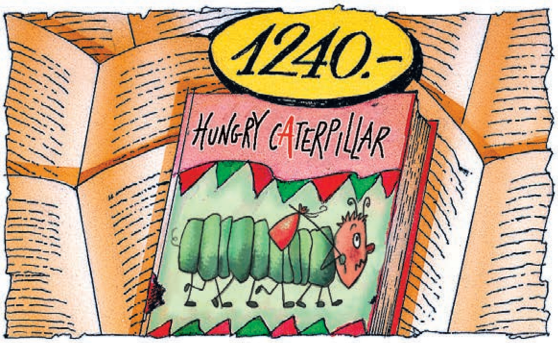
Decimals

Integers

Orientation

1 Natural numbers

13. Changes of the product



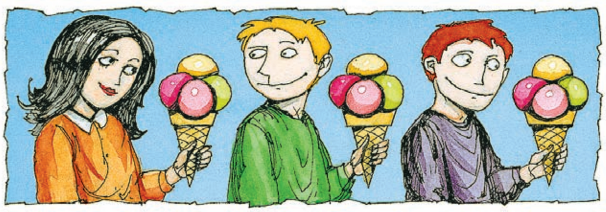
The school librarian would like to buy 10 Hungry Caterpillar novels. The book costs 1240 Ft now, but it will be half price next week. How much would 10 books cost today? How much would they cost next week?

Now: 10 · 1240 Ft = 12 400 Ft. *Half price:* 10 · (1240 Ft : 2) = 10 · 620 Ft = 6200 Ft.

If one book is half price, then 10 books are also half price.

Changes in the product

Three children eat 4 scoops of ice cream each. How many scoops do they eat altogether?



Three children eat 12 scoops of ice cream altogether.

$$3 \cdot 4 = 12$$

Observe the changes in the product in the following cases.

If twice as many children eat 4 scoops each.

(2 · 3) · 4 = 6 · 4 = 24

If one of the factors is doubled (the other is unchanged), then the product also doubles.

If the same number of children eat half as much ice cream.

3 · (4 : 2) = 3 · 2 = 6

If one of the factors is halved (the other is unchanged), then the product is also halved.

If twice as many children eat half as much ice cream.

(2 · 3) · (4 : 2) = 6 · 2 = 12

If one of the factors is double and the other is halved, then the product is unchanged.

Example

Multiply it in the simplest way: *a)* $36 \cdot 25$; *b)* $68 \cdot 50$; *c)* $33 \cdot 30$.

Solution

Based on what we learned about the changes of the product:

a) $36 \cdot 25 =$ (36 : 4, 25 · 4) $= 9 \cdot 100 = 900$;

b) $68 \cdot 50 =$ (68 : 2, 50 · 2) $= 34 \cdot 100 = 3400$;

c) $33 \cdot 30 =$ (33 · 3, 30 : 3) $= 99 \cdot 10 = 990$.

Exercises

1. Do the following calculations. Be clever in your method!

a) $720 \cdot 30$; *b)* $47 \cdot 20$; *c)* $130 \cdot 200$; *d)* $250 \cdot 40$; *e)* $1800 \cdot 5$; *f)* $76 \cdot 50$.

2. Which of the following statements are true and which are false?

If in the case of a two factor product:

a) the product is multiplied by three if one factor is multiplied by three while the other is unchanged;
b) the product is unchanged if one factor increases and the other decreases;
c) the product is doubled if one factor is doubled, while the other is unchanged;
d) the product is multiplied by four if both factors are doubled;
e) the product is multiplied by six if both factors are multiplied by three.

In case of a multiplication with multiple factors

f) the product is unchanged if one of the factors is doubled, another is halved, and the rest are unchanged.

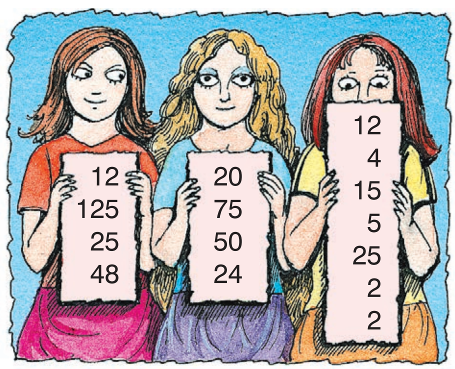
3. Do the multiplications as simply as possible.

a) $80 \cdot 25$; *b)* $50 \cdot 92$; *c)* $125 \cdot 72$; *d)* $400 \cdot 16$.

4. Do the multiplications as simply as possible.

a) $2 \cdot 28 \cdot 5$; *b)* $5 \cdot 57 \cdot 5 \cdot 4$;
c) $40 \cdot 9 \cdot 25$; *d)* $50 \cdot 5 \cdot 7 \cdot 4 \cdot 5$;
e) $72 \cdot 18 \cdot 0 \cdot 25 \cdot 50$.

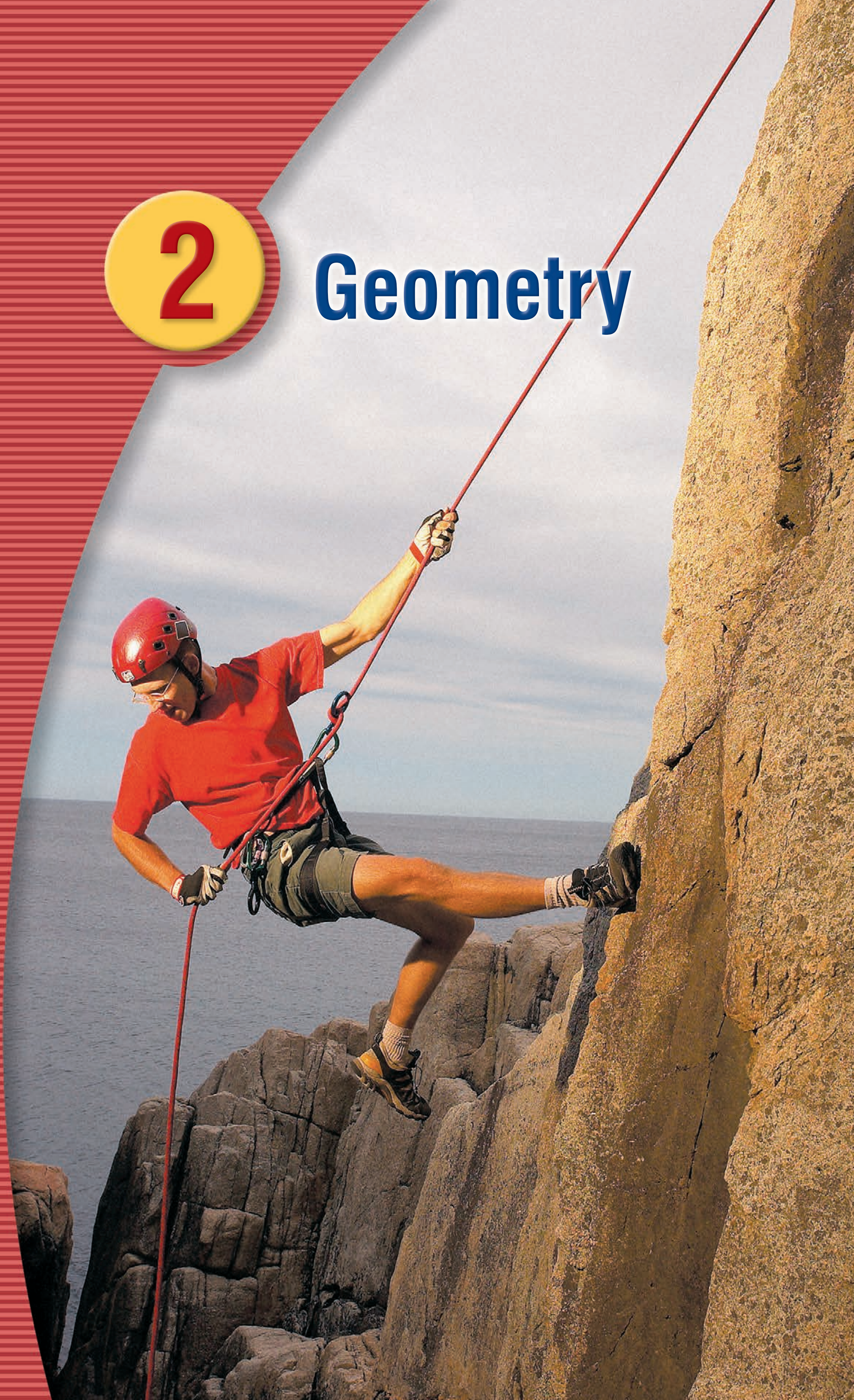
5. Sylvia, Kate and Melinda (from left to right) had to multiply the numbers on their board. Who got the largest number as a result? (All three were clever when counting.) (➡)



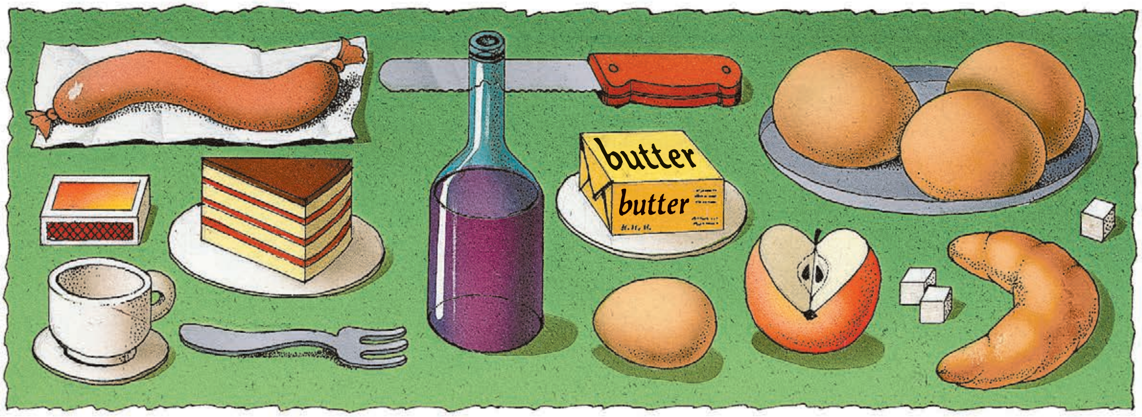
Quick

Which is more? Half a dozen dozen dozen eggs or six dozen dozen eggs? (1 dozen= 12 pieces.)

2 Geometry



7. Solids

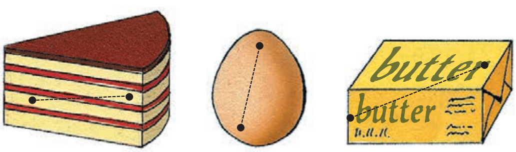


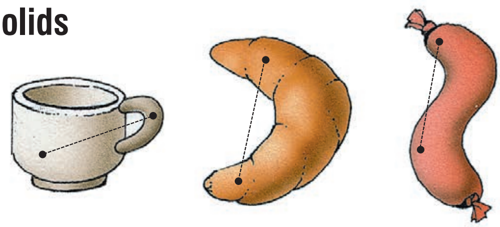
Group the solids in the image according to their characteristics.

We can group solids based on their colour, shape, material, etc.

Solids – like plane figures – can be grouped according to whether they are **convex** or **concave.**

Convex solids, concave solids



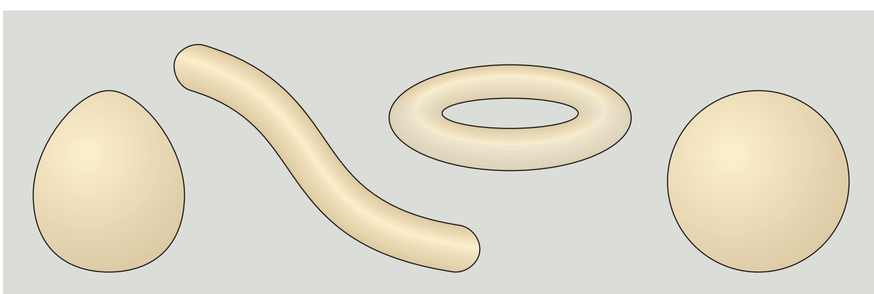


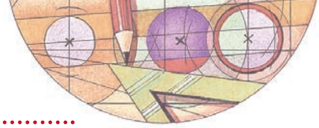
A solid is convex if for any two points, the line segment connecting them lies wholly within the solid.

A solid is concave if there exist two points whose connecting line segment does not lie wholly within the solid.

Solids can also be grouped by their surfaces.

Solids bordered by curved surfaces.





The **sphere** is a body bordered by a curved surface.

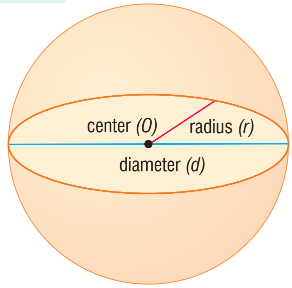
The set of points of equal distance from one point in space is a sphere.

The point is the **center (*O*)** of the sphere. The distance is the **radius (*r*).**

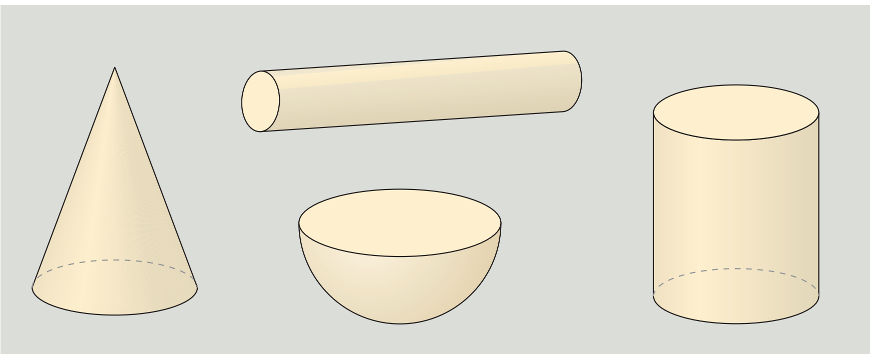
The radius of a sphere is a segment connecting the center and any point of the sphere.

The diameter of the sphere is a segment connecting two points of the sphere that passes through the center. We denote it with the letter *d.* The diameter of the sphere is twice the radius **($d = 2 \cdot r$).**

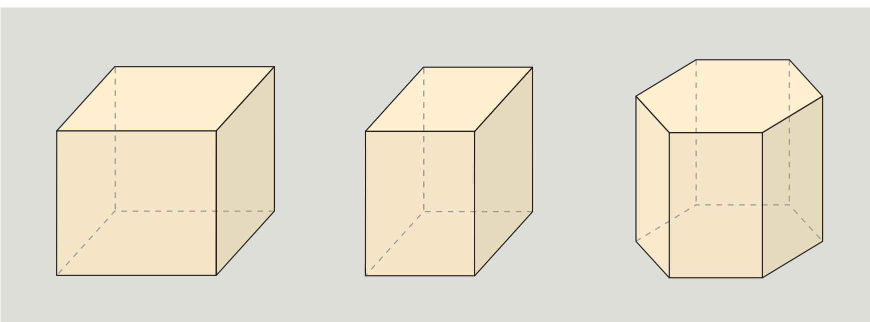
The word **sphere** can refer a **sphere's surface** or a **solid sphere**.

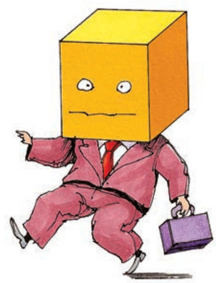


Solids bordered by curved and flat surfaces



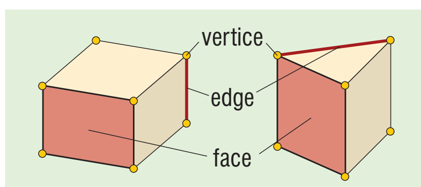
Solids bordered by flat surfaces

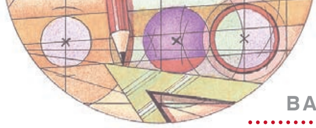




Bodies with flat surfaces are called **polyhedra**.

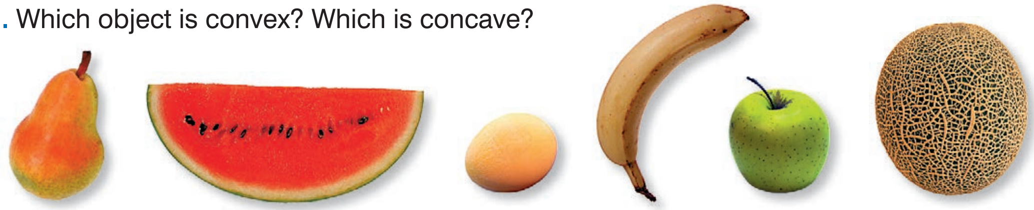
Polyhedras are bordered by **faces.**
The intersection of the faces is an **edge**.
The edges intersect at **vertices.**



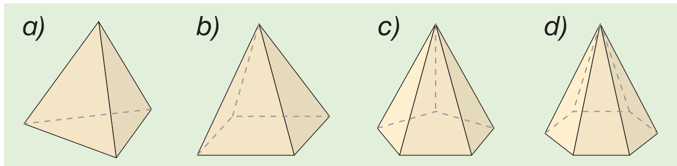


Exercises

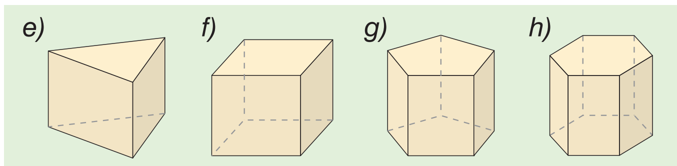
1. Which object is convex? Which is concave?



2. How many faces, edges and vertices do these solids have? How many of each polygon is the solid made of? (➡)

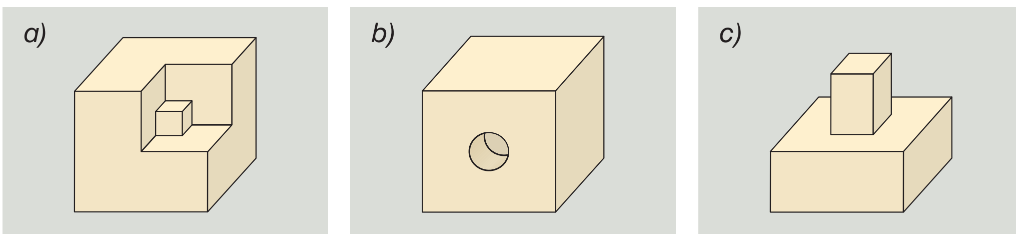


3. How many faces, edges and vertices do these solids have? How many of each polygon is the solid made of? (➡)



4. At least how many faces enclose a solid?

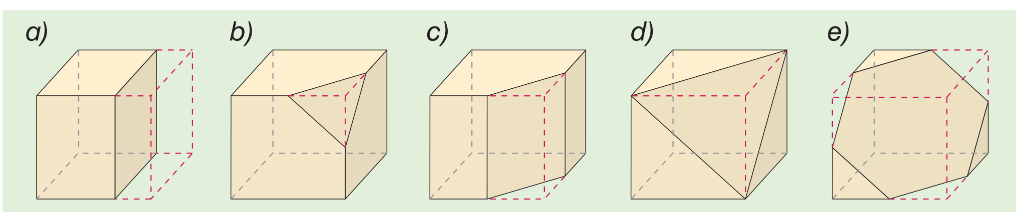
5. Is the solid missing from the cube concave or convex?



6. We cut off part of the cube with a plane.

a) How many edges, faces and vertices do the following solids have?

b) How many edges, faces and vertices do the part that have been cut off have?



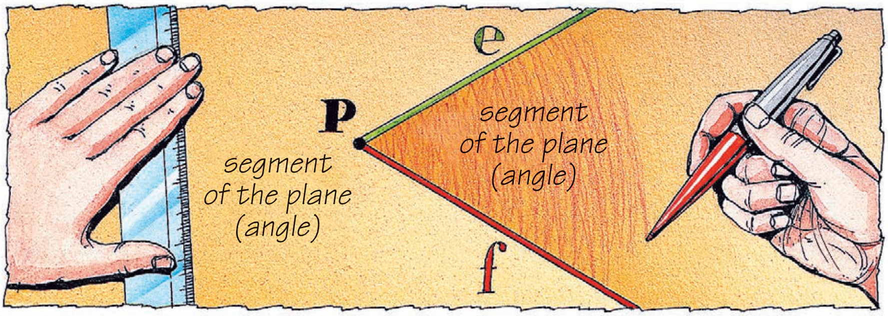
Quiz

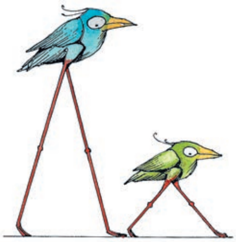
How can you make four congruent triangles using six matches, so that every side of every triangle is one matchstick long?

4 Angles

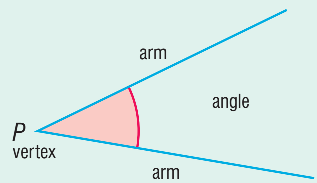
1. The concept of the angle

Draw two rays starting from the same point. These two lines divide the plane into two parts.

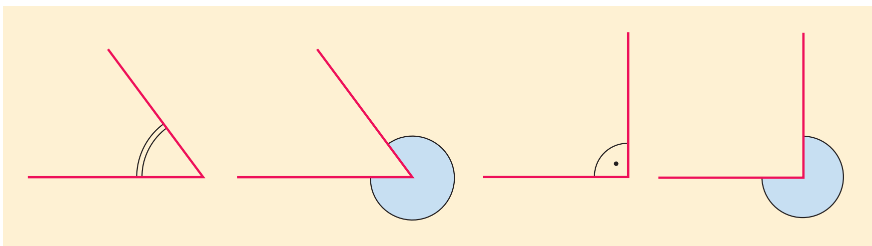




Two rays originating from the same point divide the plane into two angles. The rays are the arms of the angle, the starting point is the vertex.



On the image we label which part of the divided plane, or angle we are referring to.

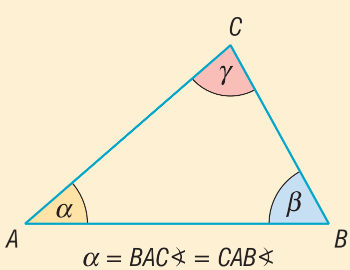


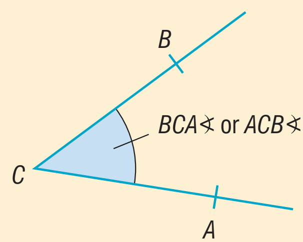
αβγδεφ

We can label angles with lower case letters of the Greek alphabet (e.g. α), or by writing the point of one arm, the vertex and a point of the other arm together (e.g. *BCA*∢). The central letter is always the vertex of the angle.

The most common letters of the Greek alphabet:

α (alpha), β (beta), γ (gamma), δ (delta), ε (epsilon), π (pi), ρ (rho), σ (sigma), φ (phi), ω (omega), η (eta)

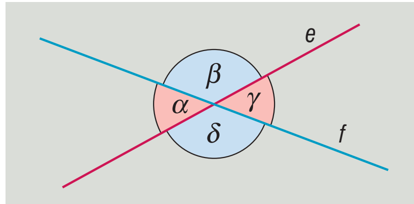




The angle of two intersecting lines

Two intersecting lines divide the plane into four parts. The following is true of the resulting angles $\alpha = \gamma$ and $\beta = \delta$.

When talking of an angle formed by intersecting lines, we are referring to the smaller or equal angle.

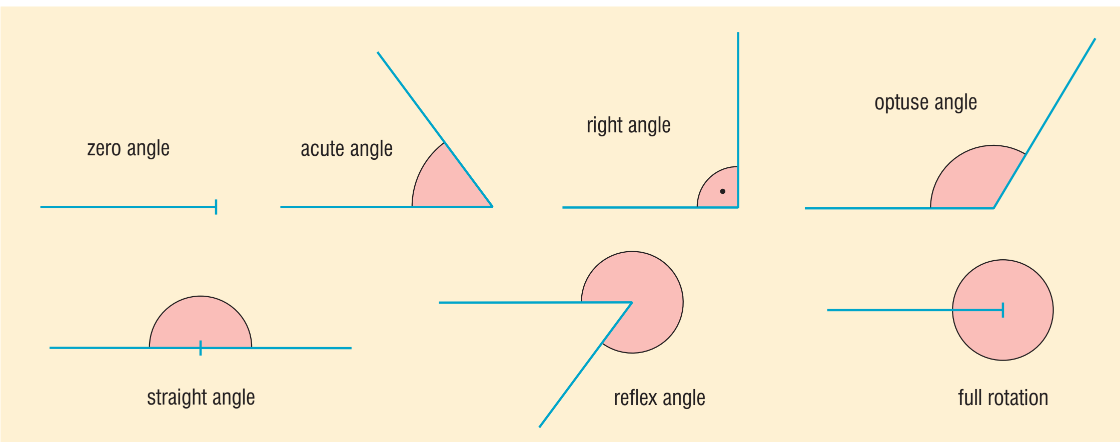


Types of angles

a) If the two arms of an angle are perpendicular we call the smaller angle **right angle.** Symbol: ⊾ or ∟.

b) If the two arms form a straight line, the angle formed is called a **straight angle.**

c) If the two arms are on top of each other the larger one is called **full rotation,** the smaller is called the **zero angle.**



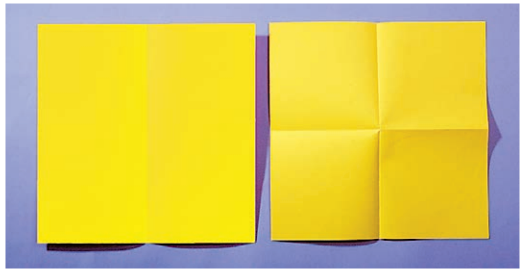
zero angle < acute angle < right angle < obtuse angle < straight angle < reflex angle < full rotation

The **acute angle** is greater than a zero angle but smaller than a right angle.
The **obtuse angle** is greater than a right angle but less than a straigh angle.
The **reflex angle** is greater than a straight angle, but less than a full rotation.
The reflex angle is concave whereas the other angles are convex.

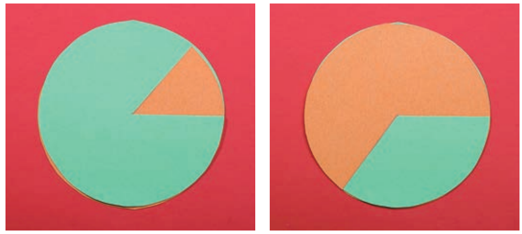
Exercises

1. Fold a piece of paper in half, then fold it in half again.

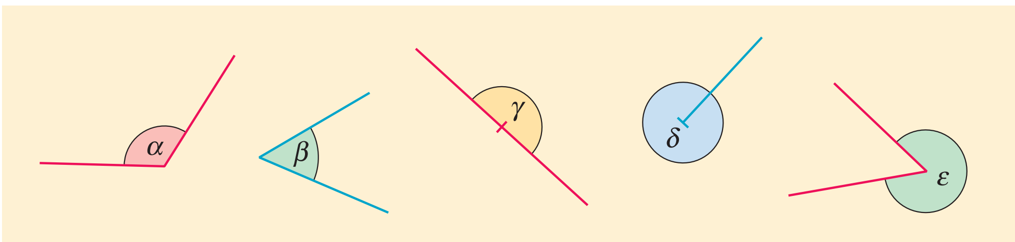
What is the position of the folds and what kind of angles do they define? (➡)



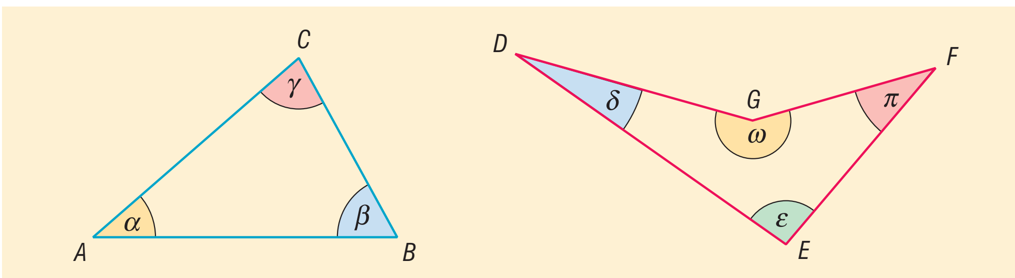
2. Make the following angle disc from two different coloured pieces of paper. Cut each one of them along a radius then put them together. Rotate one of the discs to show different angles. (➡)



3. What kind of angles are marked by the Greek letters?

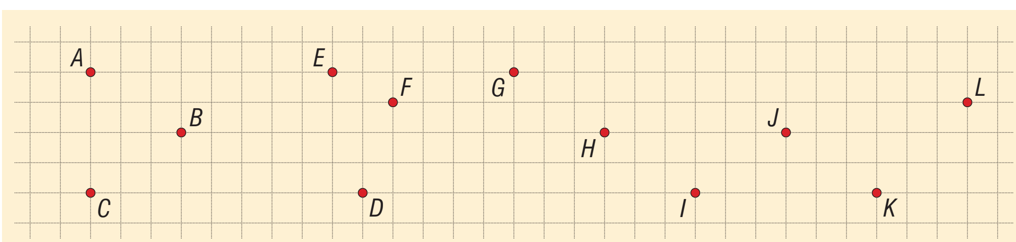


4. Write the angles marked by Greek letters using the vertices of the triangle and the quadrilateral. For example $\alpha = BAC\sphericalangle = CAB\sphericalangle$.

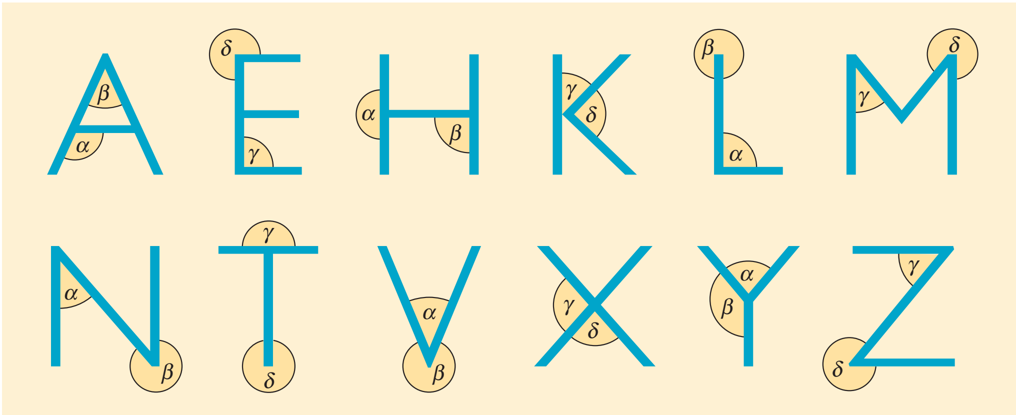


5. Draw the following angles in the image.

$\alpha = BAC\sphericalangle$; $\beta = EFD\sphericalangle$; $\gamma = GHI\sphericalangle$; $\delta = JKL\sphericalangle$.



6. What kinds of angles did we label on the letters?



7. What kind of convex angles does the small and the large hand on a clock form at:

a) 12:00; *b)* 12:30; *c)* 15:00;
d) 17:00; *e)* 17:30; *f)* 18:00;
g) 19:30; *h)* 21:00; *i)* 21:45?

8. Draw triangles that meet the following requirements

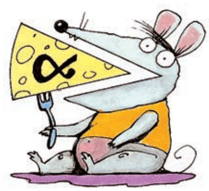
a) every angle is acute; *b)* has a right angle;
c) has a reflex angle; *d)* has two right angles.

9. Draw quadrilateral that meet the following requirements

a) all angles are right angles; *b)* has a right angle;
c) has exactly one right angle; *d)* has a reflex angle;
e) all angles are acute; *f)* has exactly two right angles.

10. What kind of angle can an acute angle be when

a) twice the size; *b)* three times the size?



11. What kind of angle can a reflex angle be

a) divided by two; *b)* divided by three?

12. Draw a quadrilateral that has

a) a reflex angle; *b)* a reflex and a right angle.

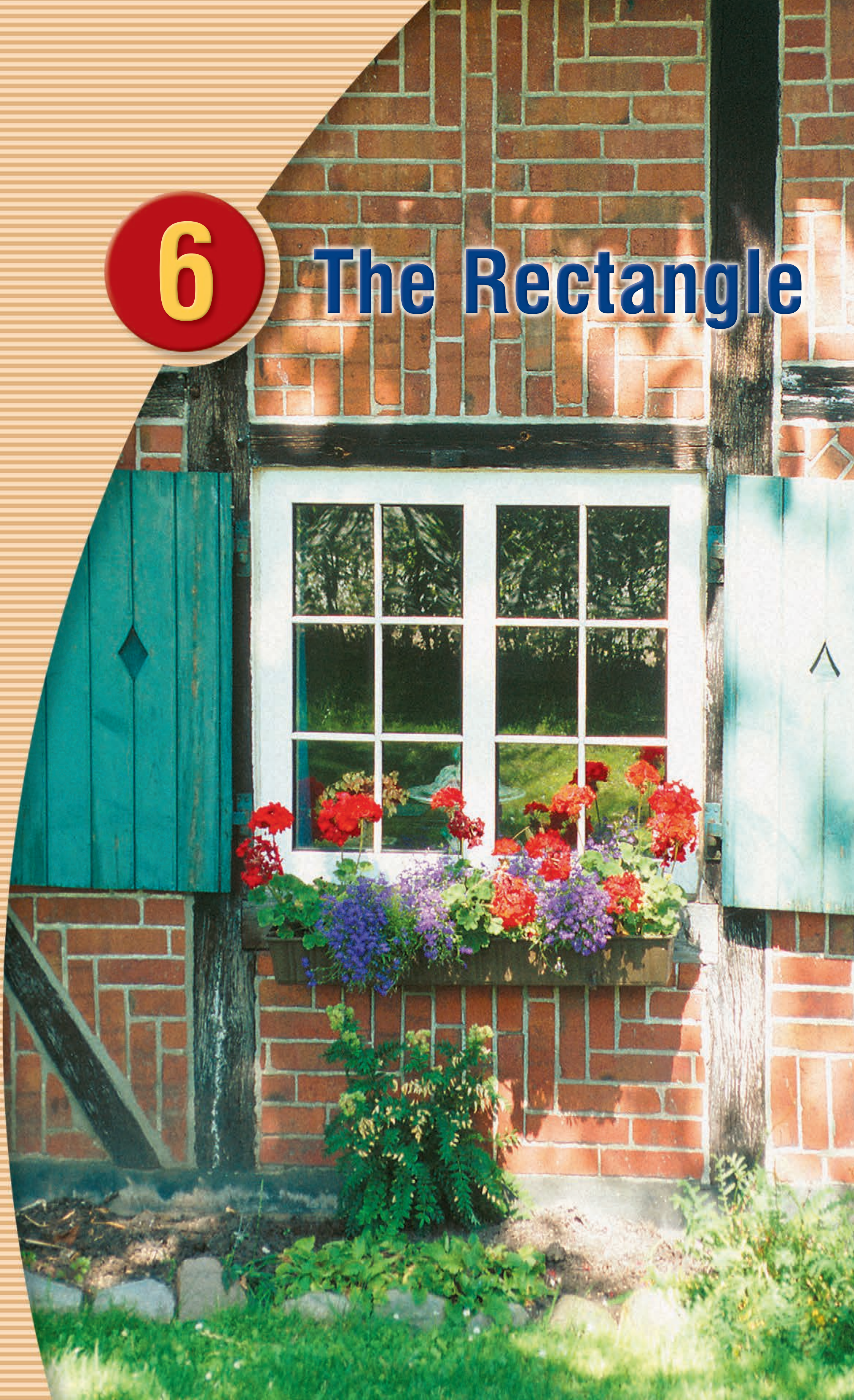
13. A boat is going north, then takes a right angle turn.
In which direction will it be going after the turn?

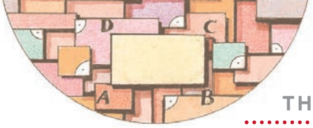
14. *a)* What angles are the cardinal and the ordinal directions compared to north?
b) Which cardinal directions are perpendicular to each other?
c) Which cardinal or ordinal directions are at acute angles to each other?
d) Which cardinal or ordinal directions are at obtuse angles to each other?

Quiz

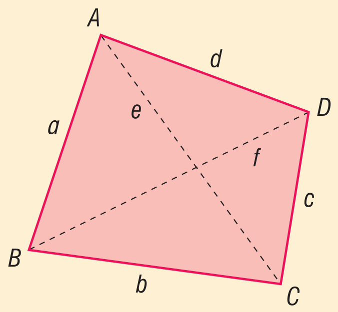
How many times does the small and the long hand of a clock overlap during the course of a day?

6 The Rectangle





1. Properties of the rectangle



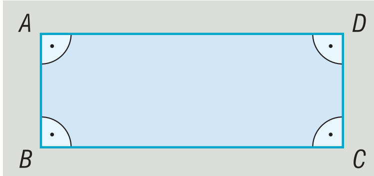
Polygons with four sides are called, **quadrilaterals.**

With quadrilaterals, we use the following terms:

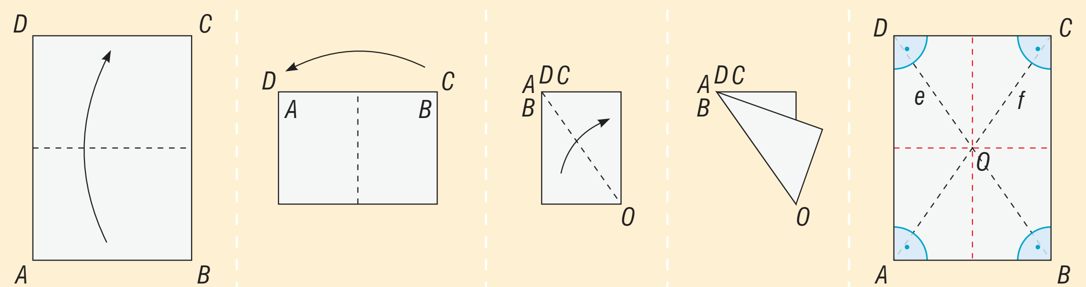
- adjacent vertices (B and C or A and D);
- opposite vertices (A and C or B and D);
- adjacent sides (a and b or a and d);
- opposite sides (a and c or b and d);
- diagonals ($e = AC$ and $f = BD$).

The rectangle

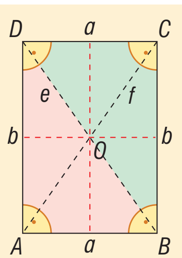
A rectangle is a quadrilateral whose adjacent sides are perpendicular to each other.



Take a **rectangular** piece of paper, and fold it in the following ways.

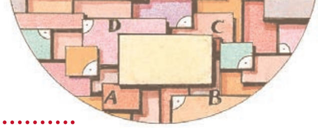


Fold point A to D, and B to C | Fold point $B(C)$ to point $A(D)$ | Fold along the line CO | Upfold it | This is what we get

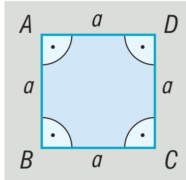


Properties of the rectangle:

- all angles are equal (90°);
- the opposite sides are parallel ($AB \parallel DC$ and $AD \parallel BC$);
- the opposite sides are equal in length ($AB = DC = a$ and $AD = BC = b$);
- the diagonals are equal ($AC = BD$; $e = f$);
- the diagonals bisect each other ($AO = CO = BO = DO$);
- it can be folded in half in two ways so that the two halves overlap (along the red lines);
- the diagonals cut it into two congruent right-angled triangles.



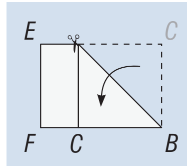
The Square



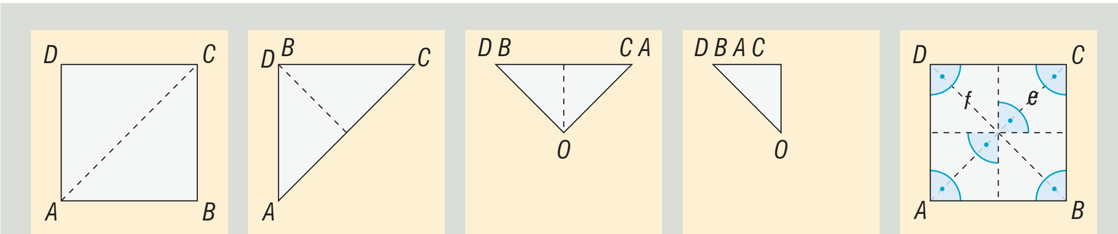
The square is a rectangle whose sides are the same length.

Make a square out of a rectangular piece of paper.

1. Fold vertex C onto side FB.
2. Cut at C parallel with EF.



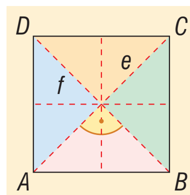
Fold the resulting square piece of paper in the following ways.



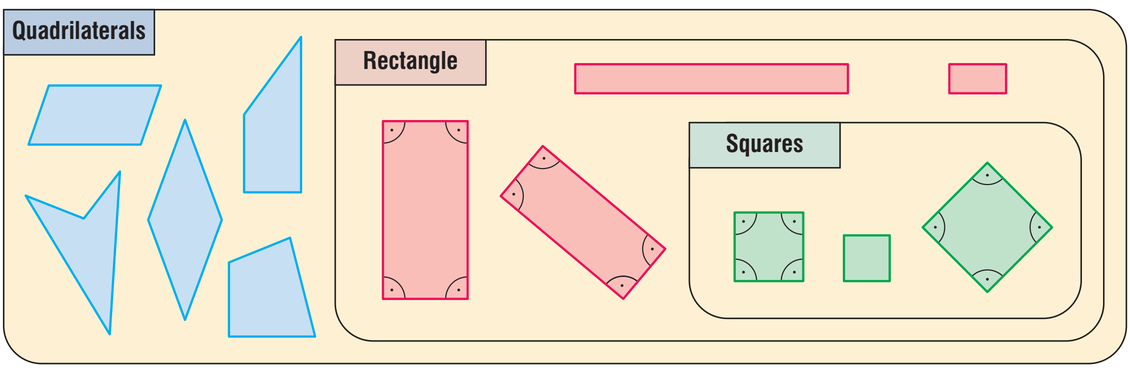
Fold in half along the line AC | Fold the vertex A to vertex C | Fold in half again | Unfold it | This is what we get

Properties of the square:

– all angles are equal (90°);
– all sides are equal;
– the diagonals are equal and bisect each other;
– the diagonals are perpendicular to each other ($AC \perp BD$; $e \perp f$);
– the diagonals divide it into four congruent triangles;
– it can be folded in half in four places so that the two halves overlap.

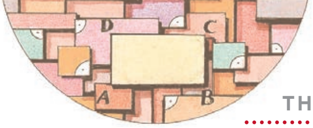


We can group quadrilaterals in the following sets:



From the image we can see that,

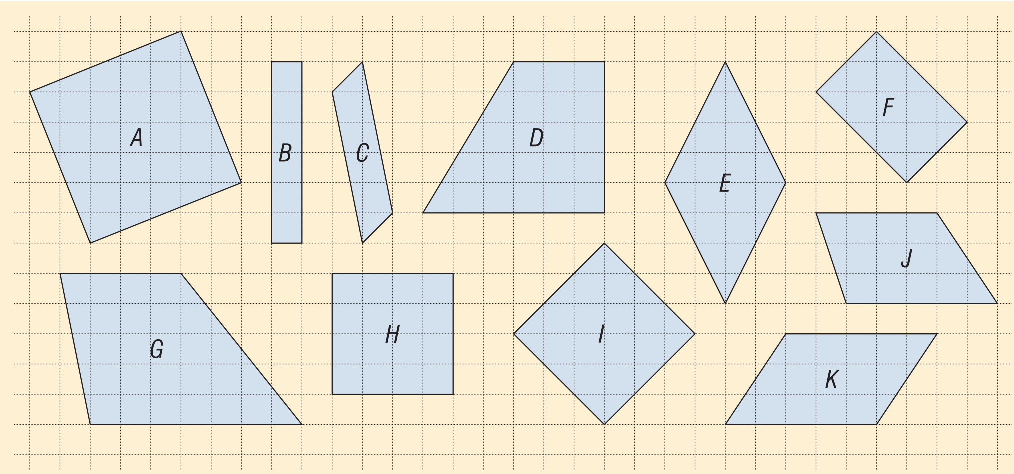
– not every quadrilateral is a rectangle;
– some rectangles are not squares.



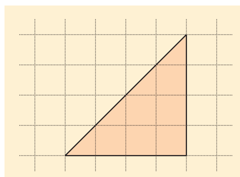
Exercises

1. List the letters of the quadrangles that are,
   a) quadrilaterals; *b)* rectangles.

   Make a set diagram and write the letters in the appropriate sets.



2. Make four triangles like the one in the image (➡)
   Use them to make a:
   a) square;
   b) a rectangle that is not a square
   Draw the compositions.



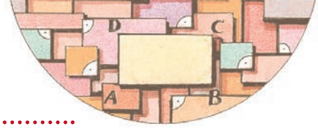
3. Divide a square into four congruent parts by folding it. Draw the folded shapes as well. Look for several solutions.

4. Can we divide a square into five congruent parts?

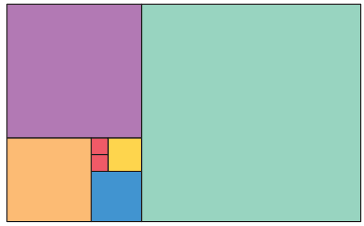
5. Divide a rectangle into four congruent parts by folding. Draw the folded shapes as well. Look for several solutions.

6. Two chopsticks are the diagonals of a quadrilateral. Put them together so that;
   a) the quadrilateral is a rectangle;
   b) the quadrilateral is a square;
   c) the two chopsticks are perpendicular, but the figure is not a square

7. Are the following statements true or false?
   a) If the diagonals of a rectangle are the same length, the rectangle is a square.
   b) If the diagonals of a rectangle are perpendicular, then it is a square.
   c) If the diagonals of a rectangle bisect each other, then it is a square.
   d) If any two sides of a rectangle are equal, it is a square.
   e) If the two sides of a rectangle are the same length it is a square.
   f) If the two adjacent sides of a rectangle are equal, it is a square.



8. How large is the side of the squares in the image below if the sides of the two smallest squares is one unit? Draw the figure on graph paper.



9. Four children cut squares and rectangles out of paper and noticed the following:

Anna: If we place two congruent squares next to each other we get a rectangle.

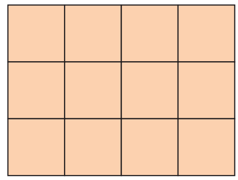
Ben: If we cut a rectangle parallel with one of its sides we get two rectangles.

Carl: If two rectangles are congruent, then placing them next to each other we definitely get a rectangle.

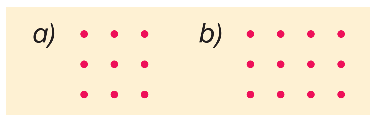
Dora: If two rectangles are not congruent we cannot place them next to each other to get a rectangle.

Who is right? Who is not? Make and draw the appropriate squares and rectangles to support your answer.

10. How many congruent rectangles are in the image? (➡)
How many rectangles are in the image?

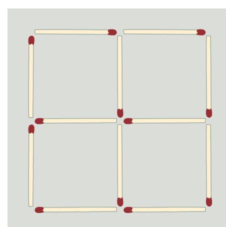


11. How many ways can you choose four of the given points so that they are the vertices of a square ? (➡)



12. With the help of graph paper, draw quadrilaterals that have exactly;
a) one; *b)* two; *c)* three; *d)* four right angles.

13. Construct the following figure from matchsticks. (➡)
a) Remove two matchsticks so that three squares remain.
b) Remove two matchsticks so that two squares remain.
You cannot overlap or place the matchsticks next to each other, they also cannot stick out from the figure.

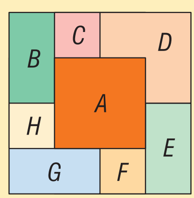


14. Based on your knowledge list sports where the playing field is rectangle shaped and ones where it is not rectangle shaped.

*15. Divide a square into (not necessarily congruent)
a) 4; *b)* 7; *c)* 9;
d) 12; *e)* 17 smaller squares.

Quiz

We placed congruent squares on the table. When we put the eighth one down, they were in the position shown in the image. In what order did we place them on the table?



8 Decimals

1. The decimal number

$\frac{5}{10}$: simple fraction;

0.5: decimal fraction

Example 1

Expand the fractions so their denominators are 10; 100; 1000.

a) $\frac{1}{2}$; *b)* $\frac{3}{4}$; *c)* $\frac{171}{125}$.

Solution

a) $\frac{1}{2} = \frac{5}{10}$; *b)* $\frac{3}{4} = \frac{75}{100}$; *c)* $\frac{171}{125} = \frac{1368}{1000} = 1\frac{368}{1000}$.

Fractions with denominators of 10; 100; 1000; can be written as decimals:

a) $\frac{5}{10} = 0.5$; *b)* $\frac{75}{100} = 0.75$; *c)* $1\frac{368}{1000} = 1.368$.

$\frac{1}{10} = 0.1$

$\frac{1}{100} = 0.01$

$\frac{1}{1000} = 0.001$

The position of decimals in the place value table

In the decimal system **one step left is ten times**, **one step right is one tenth** place value. If we keep going right after the units digit, we arrive at place values less than one. These are **tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths...**

	...	*hundreds*	*tens*	*units*	*tenths*	*hundredths*	*thousandths*	...
a)			7	3	2			
b)		5	0	8	0	6		
c)				0	9	2	5	

Expanded according to place value		Mixed number form		Decimal fraction
$7 \cdot 10 + 3 \cdot 1 + 2 \cdot \frac{1}{10}$	=	$73\frac{2}{10}$	=	73.2
$5 \cdot 100 + 0 \cdot 10 + 8 \cdot 1 + 0 \cdot \frac{1}{10} + 6 \cdot \frac{1}{100}$	=	$508\frac{6}{100}$	=	508.06
$0 \cdot 1 + 9 \cdot \frac{1}{10} + 2 \cdot \frac{1}{100} + 5 \cdot \frac{1}{1000}$	=	$\frac{925}{1000}$	=	0.925

decimal

whole number | fraction

508.06

decimal point

When writing decimal numbers the **decimal point** separates the whole number from the fraction. The digits after the decimal point are called decimals. In case of numbers greater than 0 but less than 1 we write a 0 in front of the decimal point.

On some calculators and in some countries a decimal comma is used instead of a decimal point.

Reading decimal fractions

reading decimal fraction	how we say the numbers before the decimal point	what we say for the decimal point	how we say the numbers after the decimal point
73.2	seventy-three	point	two
508.06	five hundred and eight	point	zero six
0.725	zero	point	seven two five

Exercises

1. Write the numbers in the table as decimal fractions.

...	*hundreds*	*tens*	*units*	*tenths*	*hundredths*	*thousandths*	*ten thousandths*	*hundred thousandths*	*millionths*	...
		1	2	0	3	4				
			0	0	0	7	8	5	1	
	9	7	0	1	4	5	7	9		
	8	0	5	4	0	4	4			
	2	3	4	7	5	0	5	2		

2. Write the following numbers in a place value table, then write them in decimal form.

a) $5\frac{3}{10}$; *b)* $10\frac{3}{100}$; *c)* $\frac{61}{100}$; *d)* $200\frac{6}{10}$; *e)* $50\frac{60}{100}$; *f)* $60\frac{50}{1000}$.

3. Write the following numbers in decimal fraction form:

a) 3 point 4 tenth;
b) 7 point 96 hundredths;
c) 157 point 157 thousandths;
d) 2 point 51 ten thousandths;
e) 0 point 44 millionths;
f) 10 point 6 hundred thousandths.

5. Read the following numbers. Separate the numbers according to place values.

a) 44.5; *b)* 70.03; *c)* 0.007; *d)* 123.0321; *e)* 20.00006.

6. Write the following numbers in a place value table, then as decimals.

a) 2 tens + 5 tenths;
b) 12 tens + 7 tenths + 5 hundredths;
c) 2 tens + 3 hundredths + 4 thousandths;
d) 0 units + 2 tenths + 8 hundred thousandths.

7. Read the following sum: $A \cdot 1000 + B \cdot 100 + C \cdot 10 + D \cdot 1 + E \cdot \frac{1}{10} + F \cdot \frac{1}{100} + G \cdot \frac{1}{1000}$

What numbers do the letters mean in the sum if the decimal form of the sum is;

a) 803.47; *b)* 830.4; *c)* 8030.047; *d)* 345.350; *e)* 3405.035; *f)* 3045.005?

Quiz

Write the following number with as few digits as possible 0.000003.